TRINITY COLLEGE

# Year 11 Mathematics Specialist
# Test 5 2019

Calculator Free
**Matrices**

**STUDENT'S NAME** SOLUTIONS

**DATE**: Monday 26 August **TIME:** 15 minutes **MARKS**: 18

**INSTRUCTIONS:**
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (5 marks)

Determine the $2 \times 2$ transformation matrix representing:

(a) a $45^\circ$ anticlockwise rotation about the origin [2]

$$= \begin{bmatrix} \cos 45 & -\sin 45 \\ \sin 45 & \cos 45 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

(b) a reflection in the line $y = \sqrt{3}\,x$ [3]

$$\tan\theta = \sqrt{3} \implies \theta = 60^\circ$$

$$= \begin{bmatrix} \cos 120 & \sin 120 \\ \sin 120 & -\cos 120 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

2. (3 marks)

For what value(s) of $k$ is the matrix $\begin{bmatrix} 2k & 4 \\ 16 & k-4 \end{bmatrix}$ singular?

$$(2k)(k-4) - (4)(16) = 0$$

$$2k^2 - 8k - 64 = 0$$

$$2(k^2 - 4k - 32) = 0$$

$$2(k-8)(k+4) = 0$$

$$k = 8, \quad k = -4$$

3. (3 marks)

All points on the line $y = 2x - 3$ are transformed by $\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$. Determine the equation of the image line.

$$\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} k \\ 2k-3 \end{bmatrix} = \begin{bmatrix} 5k-6 \\ k \end{bmatrix}$$

$$\left.\begin{aligned} x &= 5k-6 \\ y &= k \end{aligned}\right\} \quad x = 5y - 6$$

$$x + 6 = 5y$$

$$\frac{x+6}{5} = y$$

$$0.2x + 1.2 = y$$

4. (7 marks)

Given $A = \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 \\ -2 & 4 \end{bmatrix}$

Use the matrices shown above to determine:

(a) 5A – 3I (where I is the identity matrix) [2]

$$= \begin{bmatrix} 15 & -10 \\ 5 & -5 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & -10 \\ 5 & -8 \end{bmatrix}$$

(b) $B^{-1}$ [2]

$$\det(B) = -4 - (-2)$$
$$= -2$$

$$= \frac{1}{-2} \begin{bmatrix} 4 & -1 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & \frac{1}{2} \\ -1 & \frac{1}{2} \end{bmatrix}$$

(c) matrix C: BC = A [3]

$$C = B^{-1} A$$

$$= \begin{bmatrix} -2 & \frac{1}{2} \\ -1 & \frac{1}{2} \end{bmatrix} \times \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -5.5 & 3.5 \\ -2.5 & 1.5 \end{bmatrix}$$

TRINITY COLLEGE

# Year 11 Mathematics Specialist
# Test 5 2019

Calculator Assumed
**Matrices**

**STUDENT'S NAME** ______________________________

**DATE**: Monday 26 August **TIME:** 35 minutes **MARKS**: 35

**INSTRUCTIONS:**
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (4 marks)

Determine X where $\begin{bmatrix} 2 & 3 \\ -2 & -1 \end{bmatrix} X + \begin{bmatrix} 2 \\ -1 \end{bmatrix} = X + \begin{bmatrix} 7 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 2 & 3 \\ -2 & -1 \end{bmatrix} X - X = \begin{bmatrix} 7 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\left( \begin{bmatrix} 2 & 3 \\ -2 & -1 \end{bmatrix} - I \right) X = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ -2 & -2 \end{bmatrix} X = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 3 \\ -2 & -2 \end{bmatrix}^{-1} \times \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$X = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$$

6. (14 marks)

Consider *Figure 1* on the grid below with vertices, $P_1(-1,1), Q_1(2,0), R_1(3,3)$ and $S_1(2,4)$.

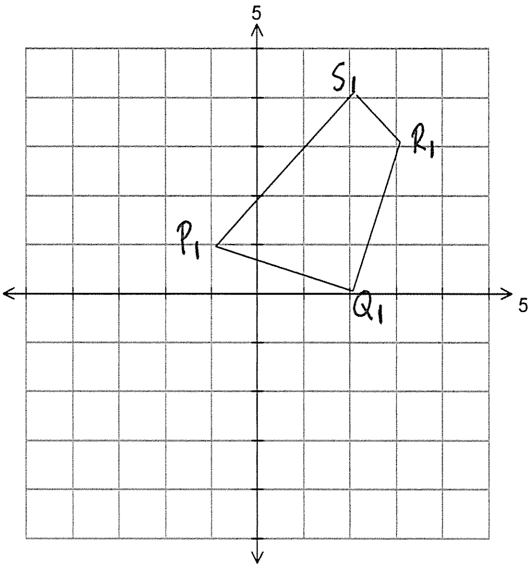


*Figure 1*

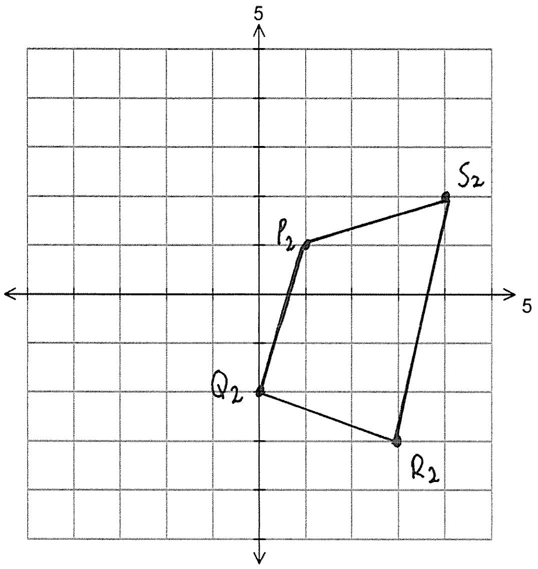


*Figure 2*

(a) The original 4 points are transformed to the points, $P_2, Q_2, R_2$ and $S_2$ by the matrix $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

(i) Plot these new points on the blank grid above on the right, indicating clearly the coordinates of each point and join them to find *Figure 2.* [4]

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} -1 & 2 & 3 & 2 \\ 1 & 0 & 3 & 4 \end{bmatrix} = \begin{array}{c} \begin{bmatrix} 1 & 0 & 3 & 4 \\ 1 & -2 & -3 & -2 \end{bmatrix} \\ P_2 \quad Q_2 \quad R_2 \quad S_2 \end{array}$$

(ii) Describe the transformation produced by the matrix A. [1]

Rotation of 90° clockwise around the origin

(b) The points $P_2, Q_2, R_2$ and $S_2$ are then transformed to the points $P_3, Q_3, R_3$ and $S_3$ by a transformation matrix that dilates the shape by a scale factor of 2 in the direction of the x-axis and a scale factor of 3 in the direction of the y-axis forming *Figure 3.* Find the matrix B that will effect this transformation and give the coordinates of P3 and Q3. [4]

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 3 & -6 \end{bmatrix}$$

$$\therefore P_3 = (2,3)$$

$$Q_3 = (0,-6)$$

(c) Write down a matrix C that transforms the original points $P_1, Q_1, R_1$ and $S_1$ directly to the points $P_3, Q_3, R_3$ and $S_3$. [2]

$$C = BA$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix}$$

(d) Write down a matrix D that transforms the points $P_3, Q_3, R_3$ and $S_3$ directly back to the original points $P_1, Q_1, R_1$ and $S_1$. [1]

$$D = \begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 0 & -\frac{1}{3} \\ \frac{1}{2} & 0 \end{bmatrix}$$

(e) If *Figure 3* has an area of 108 units$^2$, determine the area of *Figure 1.* [2]

108 = det(C) × area of figure 1

$\frac{108}{6}$ = area of figure 1

18 units$^2$ = area of figure 1

7. (9 marks)

Let $A = \begin{bmatrix} -6 & 4 \\ 5 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$.

(a) Given that $A^{-1} = kB$, determine the value of $k$. [2]

$$A^{-1} = \begin{bmatrix} 3/2 & 2 \\ 5/2 & 3 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$$

$$= \frac{1}{2} B$$

(b) The equations $4y = 6x + 4$ and $5x = 3y$ can be expressed as a matrix equation in the form $AX = C$.

(i) State the matrices $X$ and $C$. [2]

$$X = \begin{bmatrix} x \\ y \end{bmatrix} \qquad C = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

(ii) Write down a matrix equation to determine $X$ in terms of $B$ and $C$. [2]

$$AX = C$$

$$X = A^{-1}C$$

$$= \frac{1}{2}BC$$

(c) Determine the matrix $D$, if $(B - D)B = 2A$. [3]

$$(B-D)BB^{-1} = 2AB^{-1}$$

$$(B-D)I = 2A\tfrac{1}{2}A$$

$$B - D = A^2$$

$$D = B - A^2$$

$$= \begin{bmatrix} -53 & 40 \\ 50 & -23 \end{bmatrix}$$

$$A^{-1} = \tfrac{1}{2}B$$

$$2A^{-1} = B$$

$$\tfrac{1}{2}A = B^{-1}$$

8. (7 marks)

During the school vacations, the cinema puts on special programs for children. A cinema runs three sessions. The matrix S shows the attendances for Monday.

$$\mathbf{S} = \begin{array}{c} \begin{array}{cc} \textit{Adult} & \textit{Child} \end{array} \\ \begin{bmatrix} 150 & 250 \\ 160 & 250 \\ 70 & 180 \end{bmatrix} \end{array} \begin{array}{l} \\ \textit{Early} \\ \textit{Lunch} \\ \textit{Afternoon} \end{array}$$

Now the matrix of charges is given by

$$\mathbf{C} = \begin{array}{c} \$ \\ \begin{vmatrix} 9 \\ 5 \end{vmatrix} \end{array} \begin{array}{l} \\ \textit{Adults} \\ \textit{Children} \end{array}$$

(a) Determine the product matrix **R** which gives the box-office receipts for each session on Monday. [2]

$$R = \begin{bmatrix} 150 & 250 \\ 160 & 250 \\ 70 & 180 \end{bmatrix} \times \begin{bmatrix} 9 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2600 \\ 2690 \\ 1530 \end{bmatrix}$$

(b) Pre-multiply **R** by the matrix [1 1 1] to obtain the matrix **T**. [2]

$$T = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 2600 \\ 2690 \\ 1530 \end{bmatrix} = \begin{bmatrix} 6820 \end{bmatrix}$$

(c) What information is contained in **T**? [1]

total receipts fro

(d) Determine the total receipts matrix for Tuesday if the matrix of charges for Tuesday remains the same as for Monday and the attendance matrix is given below. [2]

$$\begin{array}{c} \begin{array}{cc} \textit{Adult} & \textit{Child} \end{array} \\ \begin{bmatrix} 120 & 240 \\ 150 & 210 \\ 80 & 200 \end{bmatrix} \end{array} \begin{array}{l} \\ \textit{Early} \\ \textit{Lunch} \\ \textit{Afternoon} \end{array}$$

$$\begin{bmatrix} 120 & 240 \\ 150 & 210 \\ 80 & 200 \end{bmatrix} \times \begin{bmatrix} 9 \\ 5 \end{bmatrix} = \begin{bmatrix} 2280 \\ 2400 \\ 1720 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 2280 \\ 2400 \\ 1720 \end{bmatrix} = \begin{bmatrix} 6400 \end{bmatrix}$$